Heavy-light meson decay constants with $N_f = 3$

MILC Collaboration: C. Bernard[a] T. Burch[b] S. Datta[c] C. DeTar[d] Steven Gottlieb[e]* E. Gregory,[b] Urs M. Heller[f] R. Sugar[g] and D. Toussaint[b]

[a]Department of Physics, Washington University, St. Louis, MO 63130, USA

[b]Department of Physics, University of Arizona, Tucson, AZ 85721, USA

[c]Department of Physics, Universität Bielefeld, Bielefeld, Germany

[d]Physics Department, University of Utah, Salt Lake City, UT 84112, USA

[e]Department of Physics, Indiana University, Bloomington, IN 47405, USA

[f]CSIT, Florida State University, Tallahassee, FL 32306-4130, USA

[g]Department of Physics, University of California, Santa Barbara, CA 93106, USA

During the past year the MILC Collaboration has continued its study of heavy-light meson decay constants with three dynamical quarks. Calculations have been extended to a second lattice spacing of about 0.09 fm. At this lattice spacing, there are results in the quenched approximation and for three sets of dynamical quark mass: $m_l = m_s$; $m_l = 0.4m_s$ and $m_l = 0.2m_s$, where m_l is the light mass for the u and d quarks and m_s is the strange quark mass. At the coarser lattice spacing, for which results were presented at Lattice 2001, statistics have been increased for two sets of quark masses and three additional sets of quark masses have been studied, giving a total of eight combinations used to interpolate between the quenched and chiral limits. When these calculations are completed, we can study the decay constants taking into account both chiral and continuum extrapolations.

1. INTRODUCTION

We are extending a calculation of heavy-light meson decay constants with three flavors of dynamical quarks that was begun last year [1]. In addition to increasing statistics on some runs, we have additional mass combinations for $a = 0.13$ fm and new dynamical quark runs with $a = 0.09$ fm. Runs for the coarser lattice spacing are completed, but running will continue for the finer lattice spacing, so that we can begin to understand the continuum limit.

Dynamical gauge configurations are generated using the Asqtad action [2]. For the heavy-light mesons, we use tadpole-improved clover valence quarks and operators that are improved according to the Fermilab formalism [3]. For each ensemble of dynamical quark configurations, we use five light and five heavy valence quark masses. The masses and decay constants are interpolated or extrapolated as explained below to get physically relevant values with the overall scale set by the ρ mass.

*presented by S. Gottlieb

2. PROGRESS SINCE LATTICE 01

Last year we had completed running for $a = 0.13$ fm on quenched configurations, on a two flavor ensemble, and for four combinations of light and strange quark dynamical masses. A fraction of another set was run. We have now completed eight sets of dynamical masses with $a = 0.13$ fm. Our quenched run for $a = 0.09$ fm included about 15% of the configurations there, and we were generating configurations with two sets of dynamical masses. We soon completed analysis of every-other quenched lattice and have made substantial progress on three dynamical ensembles. The table below contains a summary of our running.

dynamical $am_{u,d}/am_s$	β	configs. generated	configs. analyzed
$a = 0.13$ fm; $20^3 \times 64$			
∞/∞	8.00	408	290
$0.02/\infty$	7.20	411	411
0.40/0.40	7.35	332	324
0.20/0.20	7.15	341	341
0.10/0.10	6.96	340	340
0.05/0.05	6.85	425	425
0.04/0.05	6.83	351	347
0.03/0.05	6.81	564	563
0.02/0.05	6.79	486	486
0.01/0.05	6.76	407	399
$a = 0.09$ fm; $28^3 \times 96$			
∞/∞	8.40	417	200
0.031/0.031	7.18	336	163
0.0124/0.031	7.11	370	120
0.0062/0.031	7.09	176	48

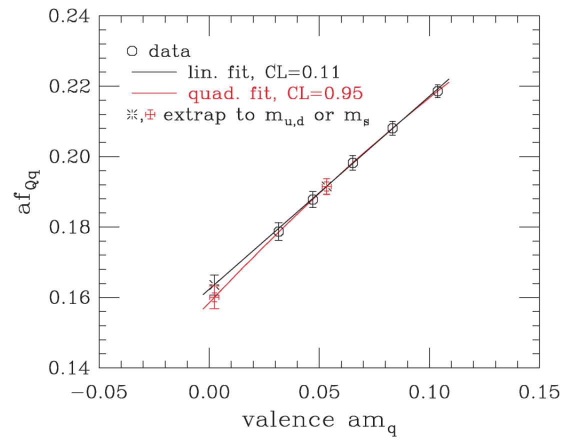

Figure 1. Chiral extrapolation of f_B showing both linear and non-linear fits for $\beta = 6.79$, $\kappa_Q = 0.08$.

3. ANALYSIS OF RESULTS

The analysis of the heavy-light decay constants involves a number of steps:

1. fit light pseudoscalar masses
2. perform chiral fit of pseudoscalar masses to determine κ_c
3. fit light vector meson masses
4. perform chiral fit of vector meson masses and determine $m_{u,d}$ and a to get physical m_π/m_ρ and m_ρ
5. determine m_s from mass of $\bar{s}s$ pseudoscalar state assuming linear chiral mass relation
6. fit heavy-light channels to determine masses and decay amplitudes
7. extrapolate or interpolate results in light quark mass to $m_{u,d}$ or m_s, respectively (see Fig. 1). One can see that the confidence level of the nonlinear fit is much better than that of the linear fit. The curvature of the former has the same sign as that expected from chiral logarithms as recently suggested [4]; however, our data has considerably less curvature than the lowest order chiral logarithms would predict, requiring higher-order terms and, perhaps, a modification of the logarithms due to 'taste' breaking or partial quenching.
8. after removal of perturbative logarithms, fit $f_{Qq}\sqrt{m_{Qq}}$ to a power series in $1/m_{Qq}$ and interpolate to B, B_s, D and D_s meson masses
9. put the perturbative logarithm back and use the heavy-light axial-vector current renormalization constant to get the renormalized decay-constant

Unfortunately, the axial-vector renormalization constant has not yet been calculated either perturbatively or nonperturbatively. We assume that in the quenched approximation our current results at $a = 0.13$ fm with an improved action agree with the continuum limit of our previous calculation using Wilson and clover quarks and the Wilson gauge action. This was explained in more detail in Ref. [1].

After the above procedure is done on each ensemble, we have a partially quenched result at a particular value of dynamical m_π/m_ρ. We then plot these results as a function of $(m_\pi/m_\rho)^2$ to perform a chiral extrapolation. This is demonstrated for f_B in Fig. 2.

It is worthwhile to plot the ratio of B_s and B meson decay constants since many of the systematic errors are common, and a good deal of the

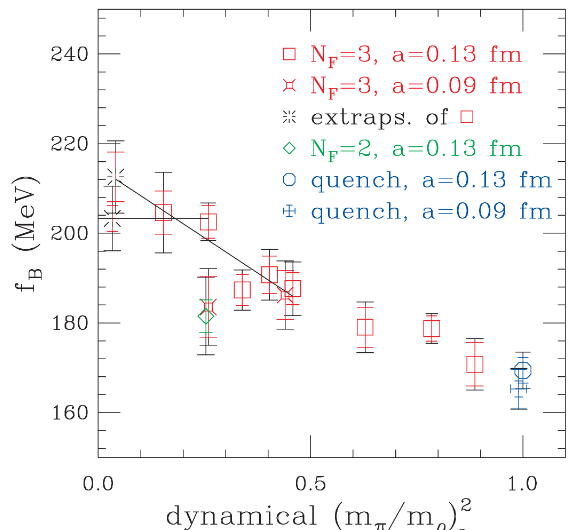

Figure 2. f_B as a function of $(m_\pi/m_\rho)^2$.

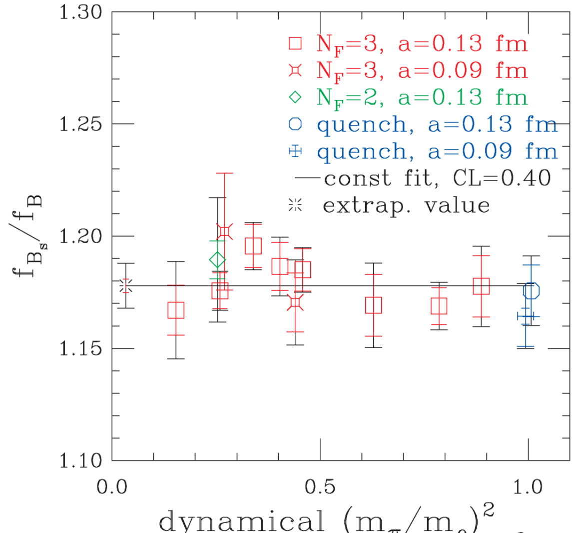

Figure 3. f_{B_s}/f_B as a function of $(m_\pi/m_\rho)^2$.

uncertainty from the renormalization constants drops out. Figure 3 shows the ratio along with a constant chiral extrapolation in the dynamical mass.

4. CONCLUSIONS

Our preliminary three flavor results and error estimates are slightly changed from last year. With higher statistics the quenched results at the finer lattice spacing are in good agreement with the coarser spacing for the B mesons. We find:

$$f_B/f_B^{\rm quench}[m_\rho{\rm scale}] = 1.23(4)(6),$$

$$f_{B_s}/f_B = 1.18(1)(^{+4}_{-1}).$$

If we take the MILC continuum quenched value (m_ρ scale) for f_B (169 MeV) as given, our value $f_B/f_B^{\rm quench} = 1.23$ gives $f_B \approx 207$ MeV with 2+1 flavors. An independent determination of f_B awaits the calculation of the axial current renormalization factor. The errors are detailed in the table below.

Better understanding of the chiral logs is necessary and may change the f_B chiral extrapolation beyond the error estimated here.

	$f_B/f_B^{\rm quench}$	f_B	f_{B_s}/f_B
prelim. result	1.23	–	1.18
stat. error	3%	3%	1%
val. χ extr. err.	3%	3%	$^{+3}_{-0}$%
dyn. χ extr. err.	2%	2%	< 1%
perturb. err.	2%	??	$\ll$1%
discret. err.	<3%	3%?	<1%

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